

Recapping what we just did: we took three facts from lecture and combined them to derive, step-by-step, interpretations for all of the regressions involving logarithms. Let's use these interpretations to fill in the worksheet part of **Section Handout 2, part 3**:

We want to see how food consumption ( $y$ ) measured in \$/year is related to household income ( $x$ ) measured in \$/year. How would we interpret each of the following regressions?

Name	Functional Form	Interpretation in Words
linear ("constant returns")	$y = \beta_0 + \beta_1 x$	Ceteris paribus, when income increases by <u>    z    </u> , food consumption increases by <u>    <math>\widehat{\beta}_1(z)</math>    </u> .
log ("decreasing returns")	$y = \beta_0 + \beta_1 \log x$	Ceteris paribus, when income increases by <u>    z percent    </u> , food consumption increases by <u>    <math>\frac{1}{100} \widehat{\beta}_1(z)</math>    </u> .
log-linear ("increasing returns")	$\log y = \beta_0 + \beta_1 x$	Ceteris paribus, when income increases by <u>    z    </u> , food consumption increases by <u>    <b><math>100\widehat{\beta}_1(z)</math> percent</b>    </u> .
log-log ("constant elasticity")	$\log y = \beta_0 + \beta_1 \log x$	Ceteris paribus, when income increases by <u>    z percent    </u> , food consumption increases by <u>    <math>\widehat{\beta}_1(z)</math> percent    </u> .

Let's do a real example with some numbers. Here I'm going to use different functional forms for regressions relating hourly wage (in \$) with years of education, using Wooldridge's data from example 2.4.

Name	Regression Results	Interpretation in Words
linear ("constant returns")	$\widehat{wage} = -0.90 + 0.54(education)$	Ceteris paribus, when education increases by <b>1 year</b> , wage changes by <b><math>0.54(1) = \\$0.54</math></b> .
log ("decreasing returns")	$\widehat{wage} = -7.46 + 5.33 \log(education)$	Ceteris paribus, when education increases by <b>10%</b> , wage increases by <b><math>\frac{1}{100} 5.33(10) = \\$0.533</math></b> .
log-linear ("increasing returns")	$\log(\widehat{wage}) = 0.58 + 0.08(education)$	Ceteris paribus, when education increases by <b>1 year</b> , wage increases by <b><math>100(0.08)(1)\% = 8\%</math></b> .
log-log ("constant elasticity")	$\log(\widehat{wage}) = -0.44 + 0.83 \log(education)$	Ceteris paribus, when education increases by <b>10%</b> , wage increases by <b><math>0.83(10)\% = 8.3\%</math></b> .